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HYDRAULIC APPARATUS FOR ENGINEERING COMPUTATIONS

by

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FOREWORD

Material presented herein was translated by Mr. Dimitry Vergun of the Massachusetts Institute of Technology, Division of Industrial Cooperation, as part of Contract No. DA-19-016-Eng-2743, Design, Construction, and Exploratory Operation of an Hydraulic Analog of the Frost Penetration Problem.

The translated report pertains to an hydraulic apparatus for making engineering calculations. This machine is similar in principles of design and operation to the one which is being constructed under the terms of that contract by the Massachusetts Institute of Technology for the New England Division, Corps of Engineers.

HYDRAULIC APPARATUS FOR ENGINEERING COMPUTATIONS

The method of computation using the hydraulic apparatus developed by the author is based on the application of principles of hydraulic analogies in correlation with partial use of the principles of computation based on finite differences. The application of these hydraulic computers, as will be shown further on, allows one to find the approximate numerical solutions for a whole class of differential equations. Therefore, these machines might be called hydraulic integrators. Insofar as the application concerns itself with the solution of differential equations, it is possible to use this method to solve problems in many different fields of technology. To approach the problem more concretely, and for clarity of presentation, we shall first look at the applications of thermodynamics to civil engineering and the problems that can now be solved by this method.

In the field of civil engineering thermodynamics, we would like to state the problem in the following manner: we have in space some body, that is, some construction of any given form; we know its thermodynamic characteristics, considering its material composition; and we know the initial distribution of temperatures. We are also given the temperature field at the time, t_0 , and the conditions of the effects on the body of surrounding bodies in space and time, and finally, if we have heat sources or sinks in the body itself, then we are given the laws describing these sources or sinks in quantitative form. We are interested then in defining the temperature field in the body under observation for any given moment

of time. In view of the fact that this requires very many variables and parameters which are parts of differential equations of the second order, the problem in this form is very difficult to solve mathematically. We will now try to approach the problem by our method. At first we will look at the simplest case: the movement of heat in one direction. As is known in this case, if we have a homogeneous material, and there are no sources of internal heat loss or heat gain, the process of the movement of the heat is defined by the differential equation of Fourier:

$$\frac{\partial T}{\partial t} = a \frac{\partial^2 T}{\partial x^2} .$$

It should be noted that the solution of this equation is extremely difficult and long. Ernst Schmidt, in an effort to find an easier solution of the problem, proposed to use the method of finite differences, and worked out the solution by means of graphics.

(Insert by translator as summary of next few paragraphs, lines 1 to 25, page 54.)

⁹⁰ The author elaborates on Schmidt's method noting that, despite the amount of work required for its solution, it has found frequent application since very often it is much easier than solving Fourier's equation. However, the amount of work involved in Schmidt's method is proportional to about the cube of the number of layers involved.

⁹¹ Then the author mentions his own method, i.e., hydraulic analog, which appears to be very similar to ours, the method being used in the hydraulic analog computer development work at the Massachusetts Institute of Technology.

⁸⁶ The author then notes that, in his method, the differentials of space are replaced by finite increments, but that differentials of time and temperature remain as differentials. Then he gives equations (1) and (2):

$$Q = k (T_n - T_{n-1}), \quad (1)$$

$$\Delta Q_o = c \Delta T, \quad (2)$$

referring to heat flow, see Figure 1; and (1') and (2') referring to water flow in an hydraulic analog, see Figure 2 :

$$Q = q (h_n - h_{n-1}), \quad (1')$$

$$\Delta Q_o = c_w \cdot \Delta h. \quad (2')$$

"In equation (1') the coefficient of proportionality, q , is the amount of liquid which flows from one element into the other in a unit of time with a difference of level of 1 **centimeter**. The first example which the author gives of the use of his method is shown in Figure 3, which represents a wall composed of many different layers and subjected to temperatures fluctuating in an arbitrary manner on both the inside and the outside. Figure 4 represents the hydraulic analogy to the situation represented in Figure 3.

"The first paragraph on page 56 describes the method of using various time constants in the system, and the author says that this can be better learned by experience. In the last two paragraphs on page 56

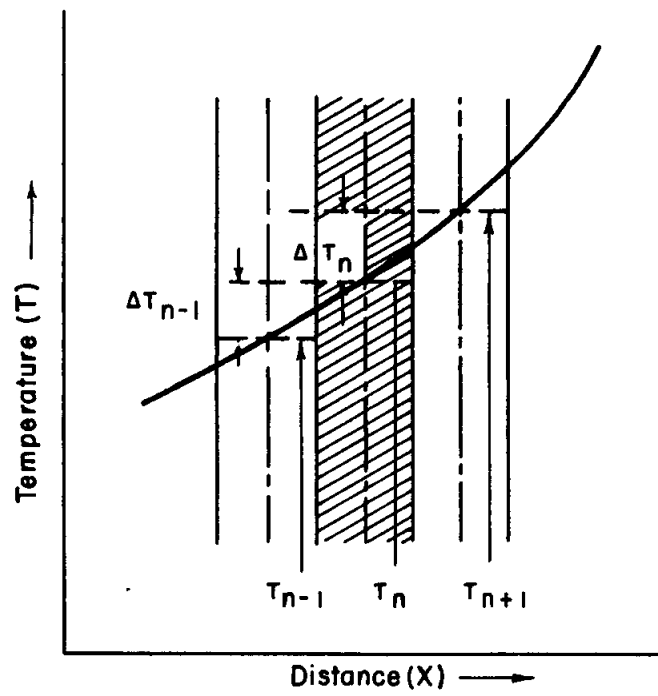


Fig. 1

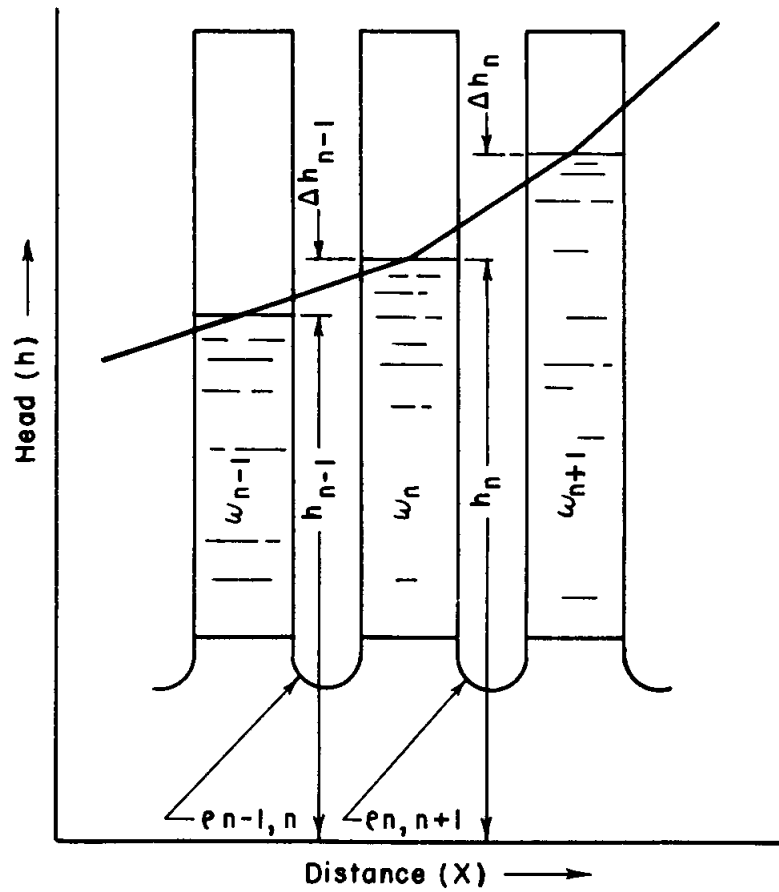


Fig. 2

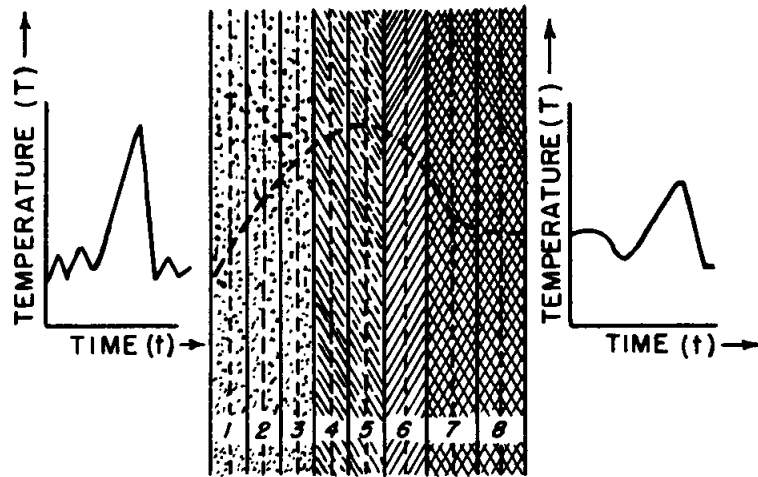


Fig. 3

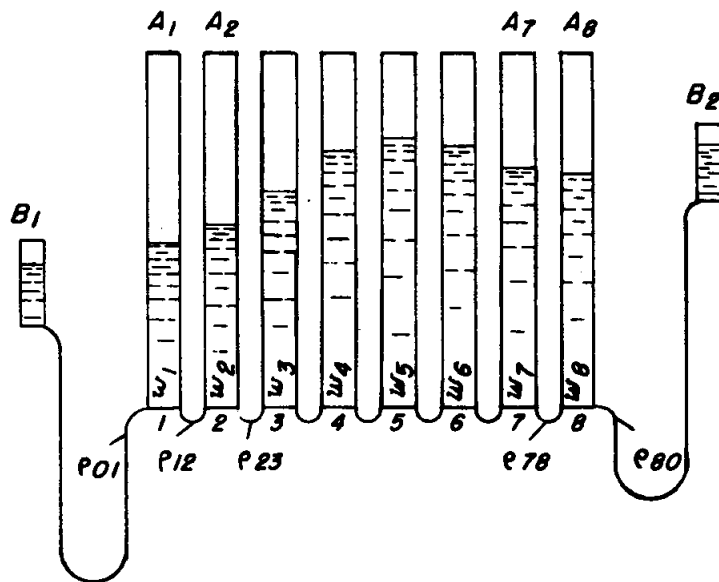


Fig. 4

and the first one of page 57 he describes the operation of 1-dimensional, 2-dimensional, and 3-dimensional analogies, as shown in Figure 5 a, b, and c, respectively. Boundary conditions are considered next. The applications of these to the 1-, 2-, and 3-dimensional problems are similar enough so that he discusses all three cases at the same time." (End of insert by translator.)

Slightly more complicated is the problem of latent heat. First, we will look at the problem as it concerns the heat of formation of ice. In the case of the freezing of water in ground, at 0°C., we have a certain amount of heat lost; when the ice melts, the heat is regained. These processes must be simulated by adding or subtracting amounts of liquid from the control receptacle. For the automatic handling of this problem the author has constructed a device with a thin-walled floating receiver which is shown schematically in Figure 6.

The thin-walled float, D, rides in chamber C. The level of liquid in chamber C is maintained constant by the aid of a special type of overflow. The interior reservoir of chamber D is connected by a rubber tube with the chamber A. The level in chamber A cannot rise or fall past 0, if we assign 0 to some level, without gaining or losing a volume of liquid equal to the volume of the reservoir in chamber D. This liquid transfer occurs while the level 0-0 remains constant.

Next we will generalize this method beyond its application to heat transfer by looking at the differential equation (Eq. 3):

$$\frac{\partial I}{\partial t} = \frac{\partial}{\partial x} \left(a \frac{\partial I}{\partial x} \right) + \frac{\partial}{\partial y} \left(b \frac{\partial I}{\partial y} \right) + \frac{\partial}{\partial z} \left(c \frac{\partial I}{\partial z} \right) + \frac{q}{v}. \quad (3)$$

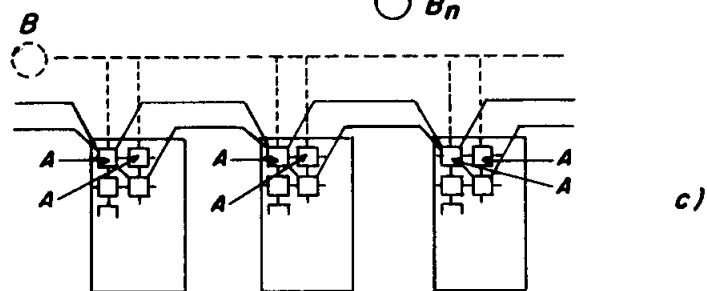
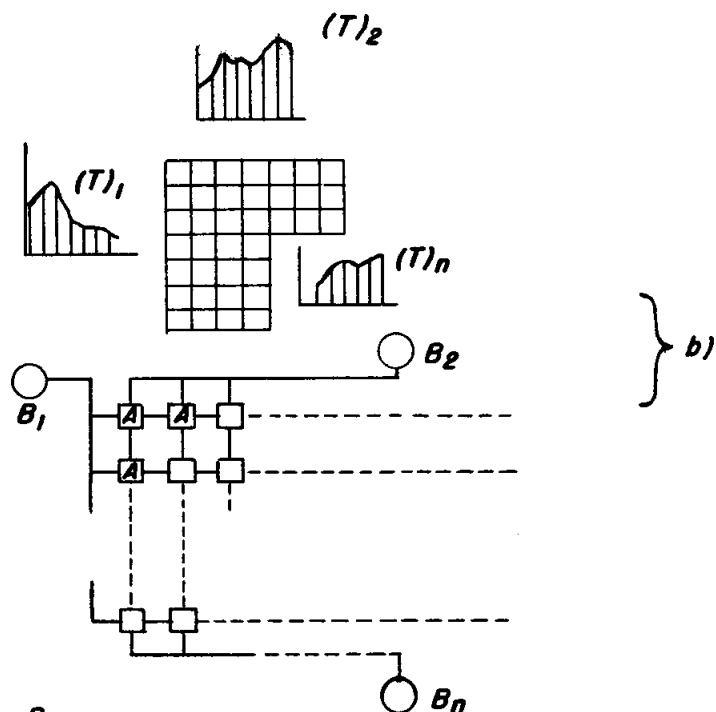
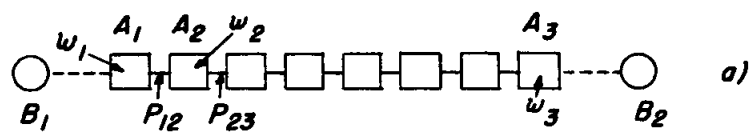


Fig. 5

In this equation x, y, z, t are the independent variables; a, b, c are known positive functions of x, y, z (they may be discontinuous); Φ is a known function of x, y, z , and t , and sometimes also of T . If we divide the region of interest into elementary volumes, $\Delta v = (\Delta x) \cdot (\Delta y) \cdot (\Delta z)$ of finite lengths, then, referring to Figure 7, equation (3) becomes equation (4):

$$\frac{\partial T}{\partial t} = \frac{1}{\Delta v_{npr}} \left\{ \left(\frac{\Delta T_n}{R_{n,n+1}} - \frac{\Delta T_{n-1}}{R_{n-1,n}} \right)_x + \left(\frac{\Delta T_p}{R_{p,p+1}} - \frac{\Delta T_{p-1}}{R_{p-1,p}} \right)_y + \left(\frac{\Delta T_r}{R_{r,r+1}} - \frac{\Delta T_{r-1}}{R_{r-1,r}} \right)_z \right\} + \Phi, \quad (4)$$

where:

$$\Delta v_{npr} = \Delta x_n \cdot \Delta y_p \cdot \Delta z_r,$$

$$R_{n,n+1} = \frac{\frac{\Delta x_n}{2a_n} + \frac{\Delta x_{n+1}}{2a_{n+1}}}{\Delta y_p \cdot \Delta z_r},$$

$$R_{n-1,n} = \frac{\frac{\Delta x_{n-1}}{2a_{n-1}} + \frac{\Delta x_n}{2a_n}}{\Delta y_p \cdot \Delta z_r},$$

and: n, p, r are the values of the given elementary volume on axes x, y, z , respectively.

Quantities R represent thermal resistances between centers of neighboring elementary volumes and are very easily simulated. If we have a system of receptacles, connected along three dimensions (such as in Fig.

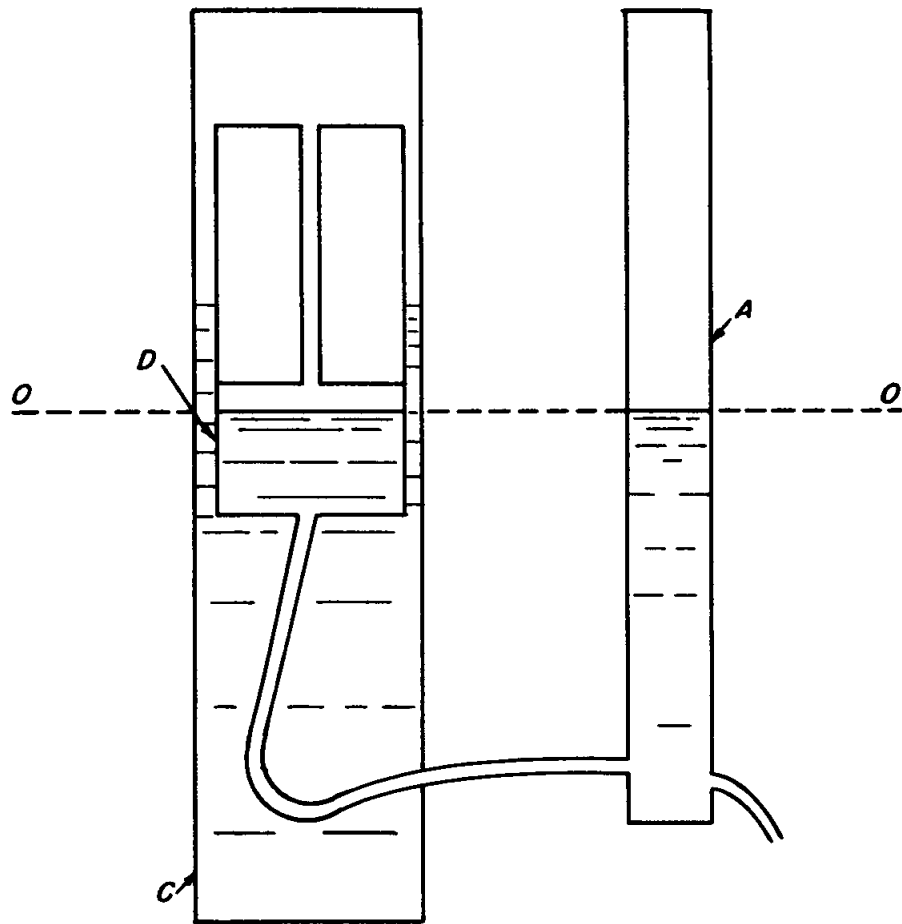


Fig. 6

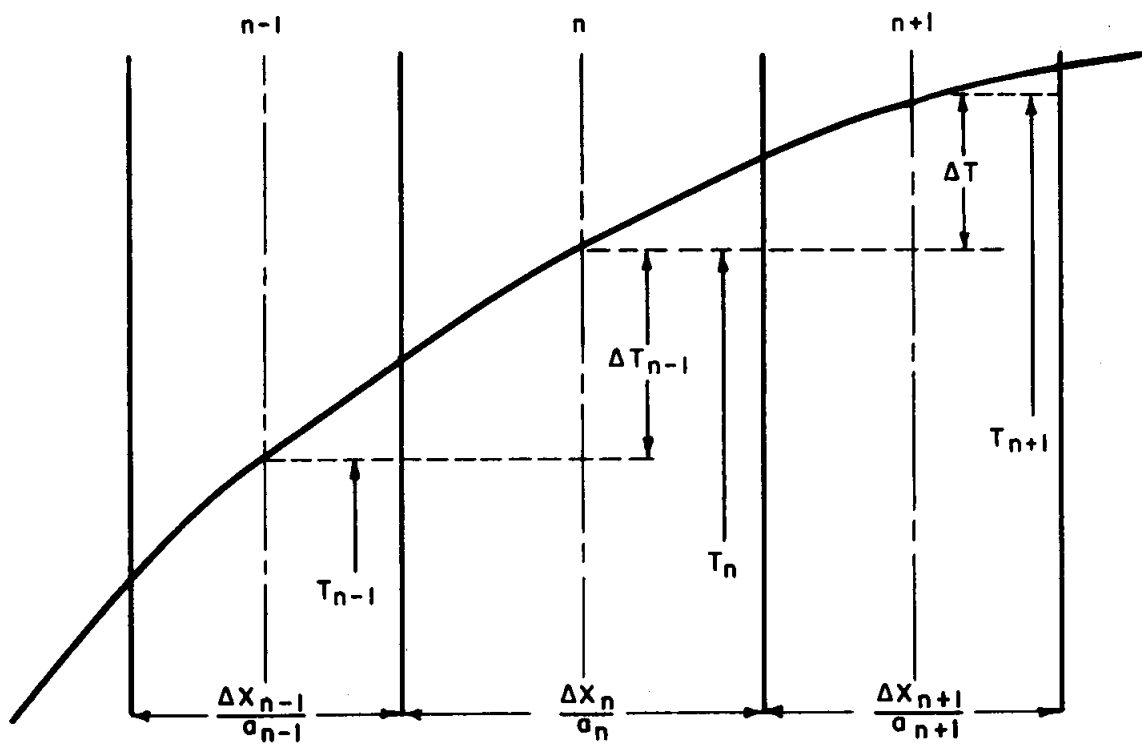


Fig. 7

5c), then the process of the changing of levels in the receptacles is defined by equation (5):

$$\frac{\partial h}{\partial t} = \frac{1}{\omega_{npr}} \left\{ \left(\frac{\Delta h_n}{e_{n,n+1}} - \frac{\Delta h_{n-1}}{e_{n-1,n}} \right)_x + \left(\frac{\Delta h_p}{e_{p,p+1}} - \frac{\Delta h_{p-1}}{e_{p-1,p}} \right)_y + \left(\frac{\Delta h_r}{e_{r,r+1}} - \frac{\Delta h_{r-1}}{e_{r-1,r}} \right)_z \right\} + \dot{Q} \quad (5)$$

It is seen that equations (4) and (5) are analogous. The coefficients R and Δv are easily computed. If we consider for the system of receptacles the area of cross section of the receptacles as ω , the individual hydraulic resistances of the connecting elements as P and if we choose the relations between these elements to be the same as that which exists between Δv and R along the whole region of the elementary volumes and if we, besides that, will add liquid at a rate of flow $\dot{Q} \cdot \omega$, then the process of the changing of levels when we follow the selected changes in the boundary conditions will define for us a quantity T (temperature). Again, the quantity T will be in a time scale dependent on the chosen unit of time. In this way this method becomes universally applicable for any process which is defined by the general equation (3). In steady state processes, the partial derivative of T with respect to time will be equal to 0, and for non-steady flow, $\frac{\partial T}{\partial t}$ is not going to be equal to 0. The generality of equation (3) permits its application in diversified fields of technology. The equations of Fourier, LaPlace, Poisson, and Pockels happen to be special cases of equation (3). The author believes that, in some cases, this hydraulic method can even be

extended to problems beyond the limits of equation (3). Undoubtedly the principles of the method permit a very wide application to problems, but, in order to realize these possibilities, it is imperative to be able to create a machine which can be applied to several kinds of problems instead of to just one. In other words, the main parts will have to be subject to change which will permit the machine to be applied to different problems. If a method is found of changing the following: (a) the cross-sectional area of the receptacles, (b) the hydraulic resistances between the receptacles, (c) connecting systems, and (d) other parameters, to meet the conditions of different problems, then the range of problems which we will be able to solve will be very great. Figure 8 is a picture of an apparatus which the author built in 1936.

It is a 1-dimensional apparatus (Fig. 5a) which has incorporated in it the means to adjust several factors. In the center is a panel with manometers (about 12) for the observation of levels in the basic receptacles which are located behind the board. To the right and to the left of the panel are located supports with movable receptacles and drums. These are used for following the variations in independent variables, such as the temperature of the outside regions. Figure 9 shows the view of the apparatus from behind. The left support is close to the apparatus and on its wall is seen the movable receptacle B (see Fig. 4), the vertical displacement of which is tied in with the vertical movement of the pen which writes on the drum. The receptacle is moved by rotating the handle under the drum, Figure 8. On the drum, which is driven at a constant speed by a clock mechanism, is drawn the predetermined movement of

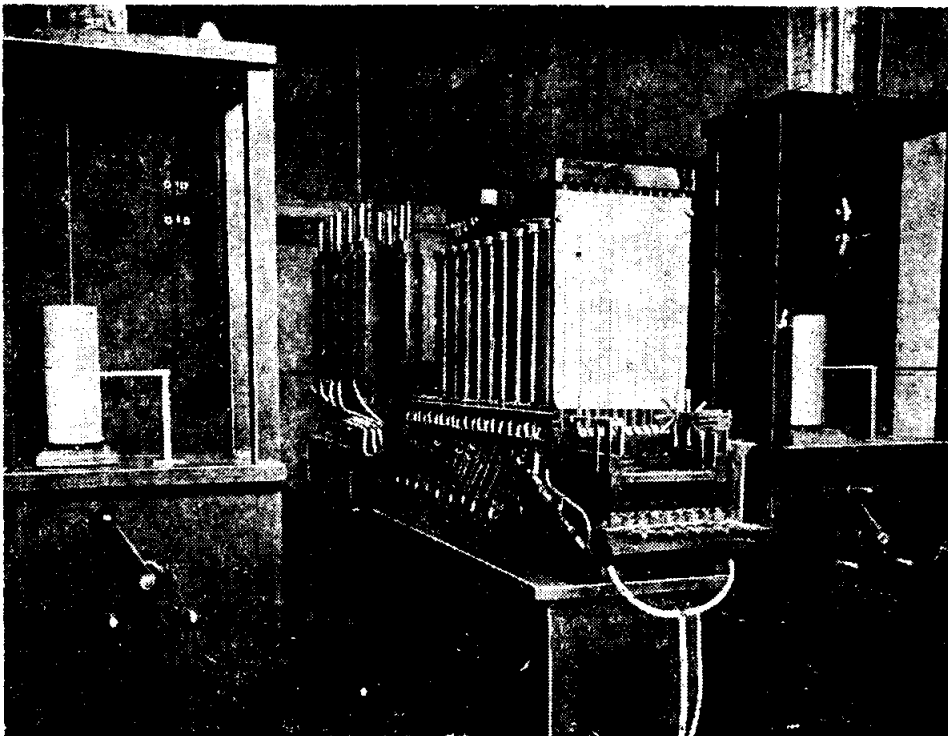


Fig. 8

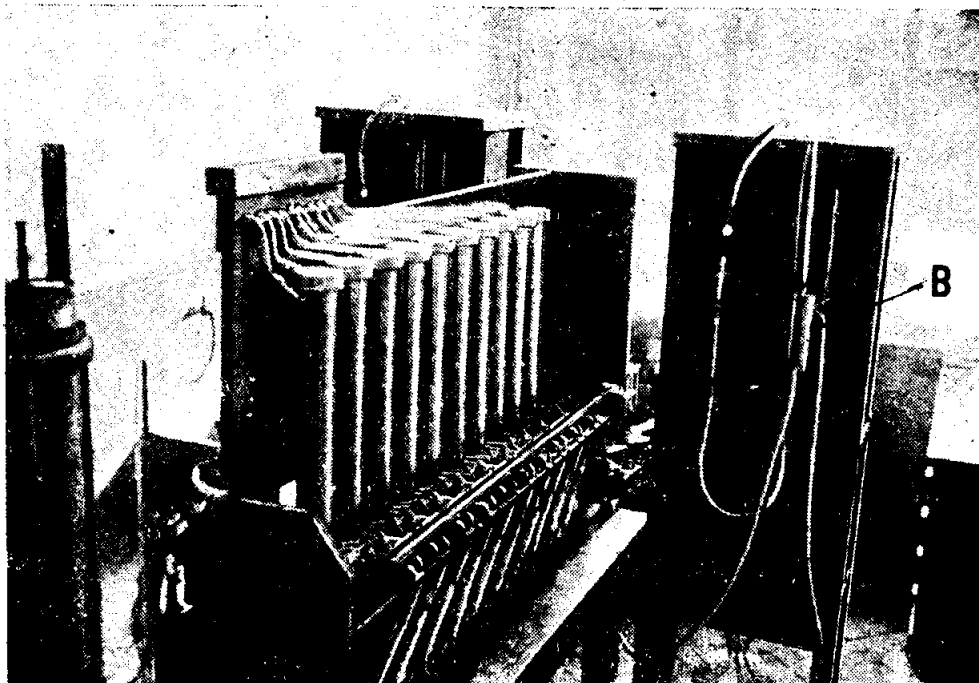


Fig. 9

temperature of the outer region. The task of the operator is to rotate the handle under the drum at a speed that will make the pen follow the course of the slowly changing curve of the outer region temperature. He must also note at times the levels in the manometers on a scale which is behind the manometers. On a panel below the manometers are seen the valves with the aid of which the whole process can be slowed down or stopped, and started again. Further below we see the cocks for the primary filling of the receptacles. This corresponds to the primary distribution of temperatures. The cross-sectional areas of the primary receptacles are changed with the requirements of the problem by means of a group of prismatic inserts with different areas of cross section which are selected to give the desired values. See Figures 10 and 11.

The individual connection tubes are changed with the aid of a group of thin glass tubes with known resistances which are shown on Figure 12. These tubes are attached to the side panel of the apparatus under the primary receptacles. At the present time it is possible to replace the glass tubes with permanent metallic elements (Fig. 13) which permit a continuous change of the individual resistances by means of changing the length of the canal.

Figure 14 shows a panel set up for a calculation involving heat generation in the body. With reference to Figure 14 the water under some changing pressure flows through the resistances to each individual receptacle and then freely flows into the corresponding receiver through tubes which are well shown on Figure 9.

Figure 15 shows receivers with floating chambers to simulate the effect of the latent heat of ice formation. In Figure 16 an

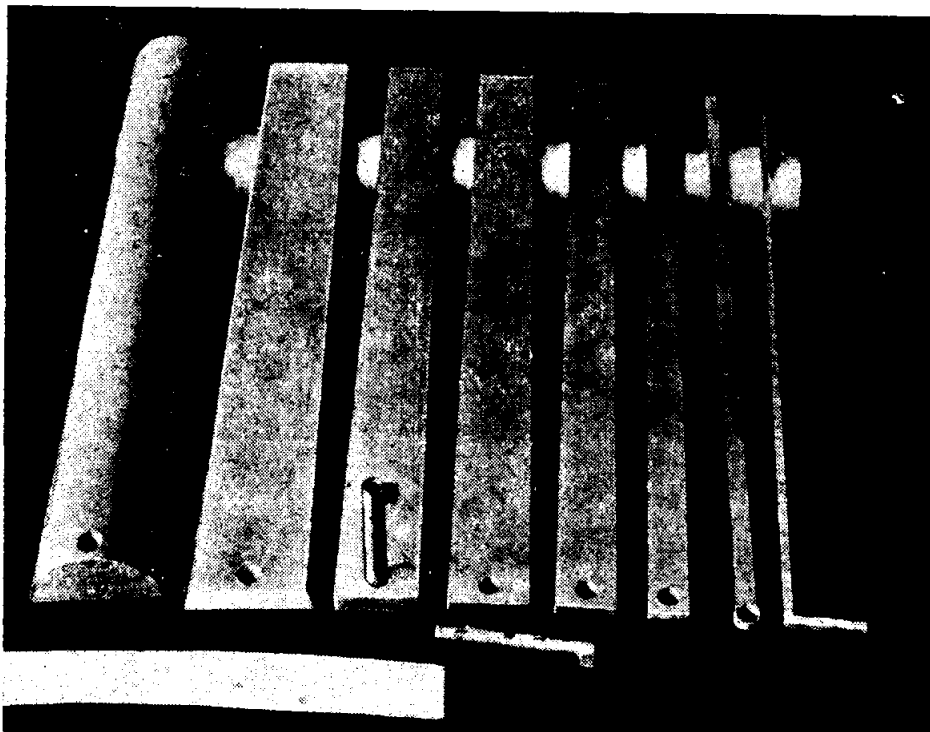


Fig. 10

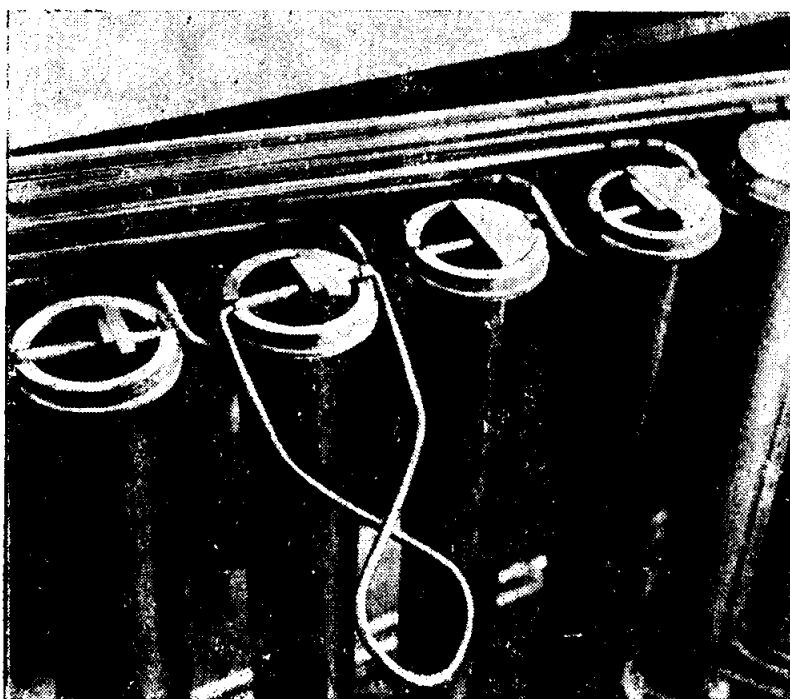


Fig. 11

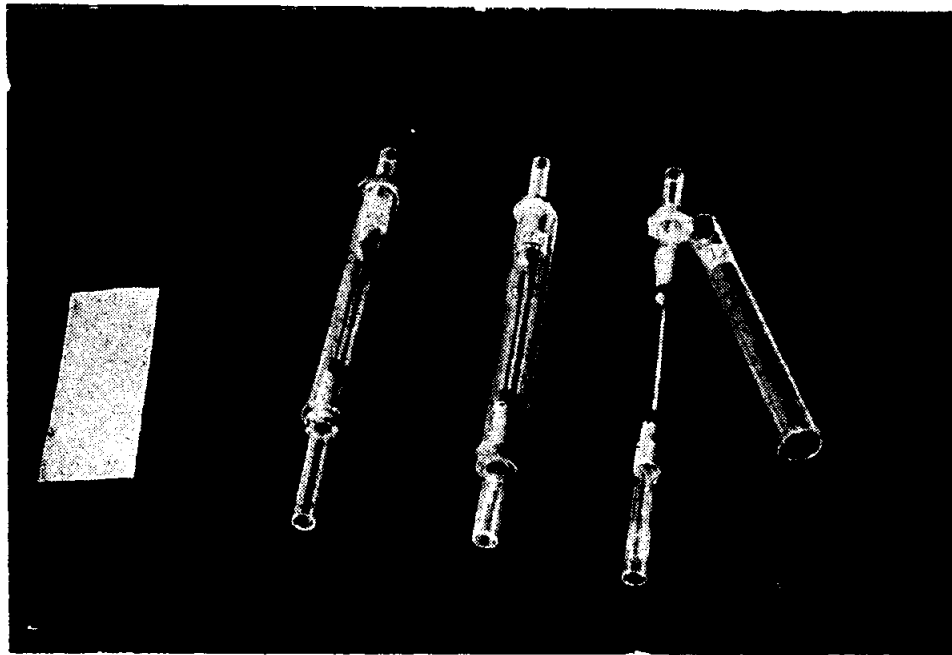


Fig. 12

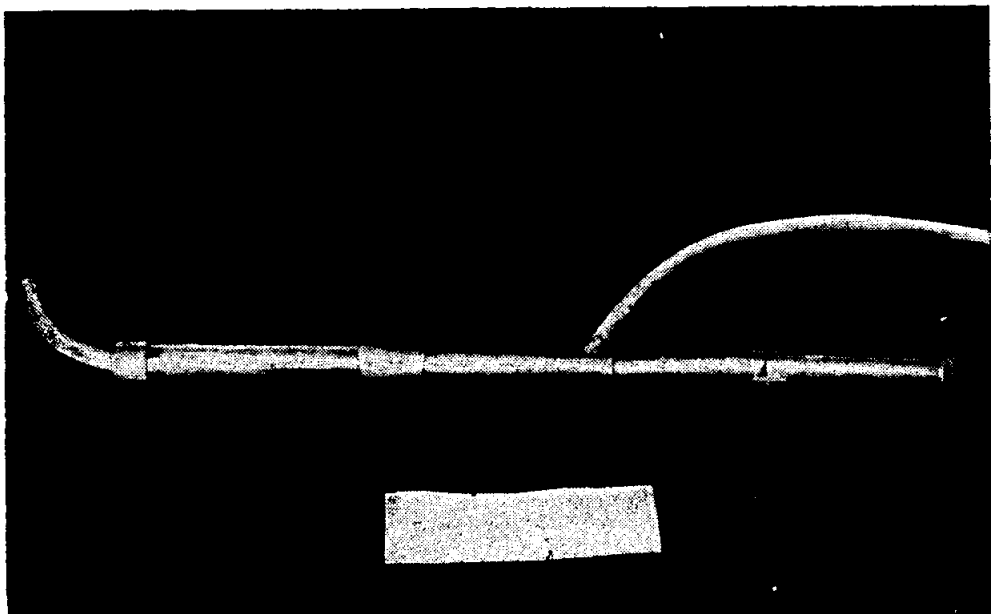


Fig. 13

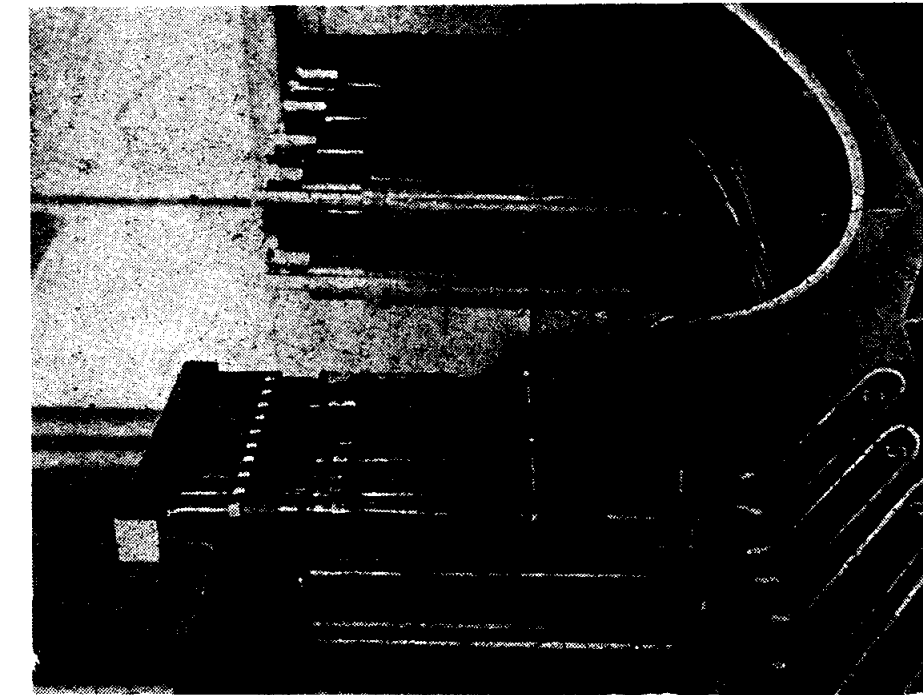


Fig. 14

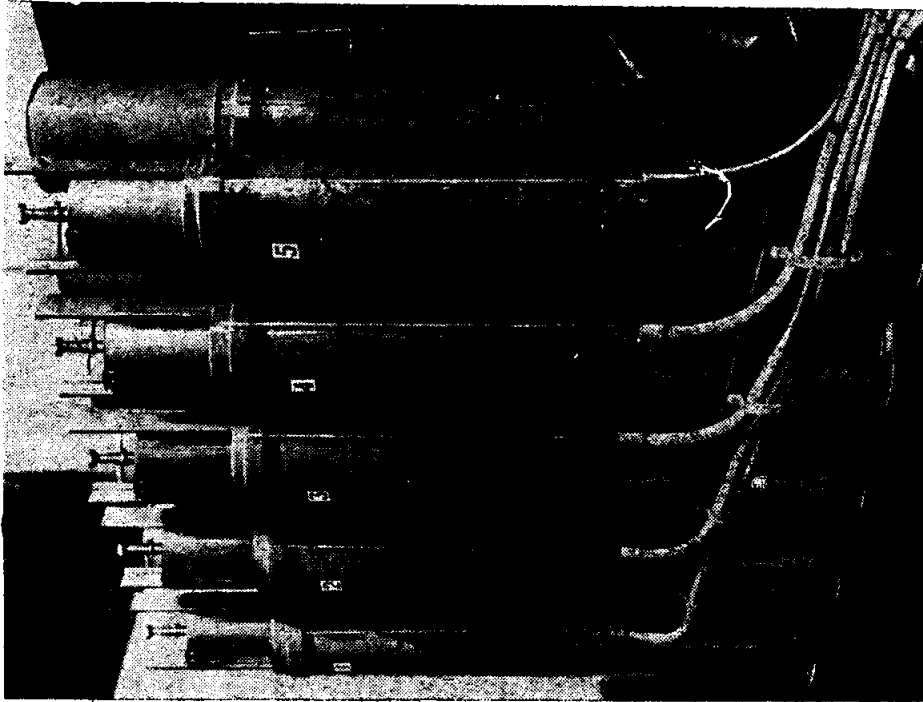


Fig. 15

enlarged photograph shows the drum for introducing the effect of the changing temperatures of surrounding space. Figure 17 is a general view of the apparatus in working order.

For a more concrete idea about the correctness of the results and for some characteristics of practical possibilities which are opened up by the utilization of this apparatus, let us show the following examples. On Figure 18 are shown the results of a calculation of the cooling of a concrete wall originally at a constant temperature ($49^{\circ}\text{C}.$) when exposed to "still" air also at a constant temperature ($+1^{\circ}\text{C}.$). This simple case allows us to compare results which are obtained by the use of this machine with results which are obtained with the help of graphical solutions. The results are very close. The slight discrepancy in the first layer is explained by the fact that for the graphical solution the first half of the first layer was considered as insulation but on the machine this assumption was not made.

Other results obtained with the machine are shown in Figure 19 for the temperature distribution and change inside of a concrete wall under the action of outside air temperatures which followed an approximately sinusoidal curve. Figure 20 shows the temperatures at different depths inside a concrete mass 5.6 meters thick which resulted from the actions of actual changes in average daily temperatures of outside air for a period of 8 months. The temperatures of the outside air are shown as a solid line, temperatures at different depths as dotted lines.

Figure 21 shows the comparison of results of a computation on the machine with the results obtained by observing the actual temperatures

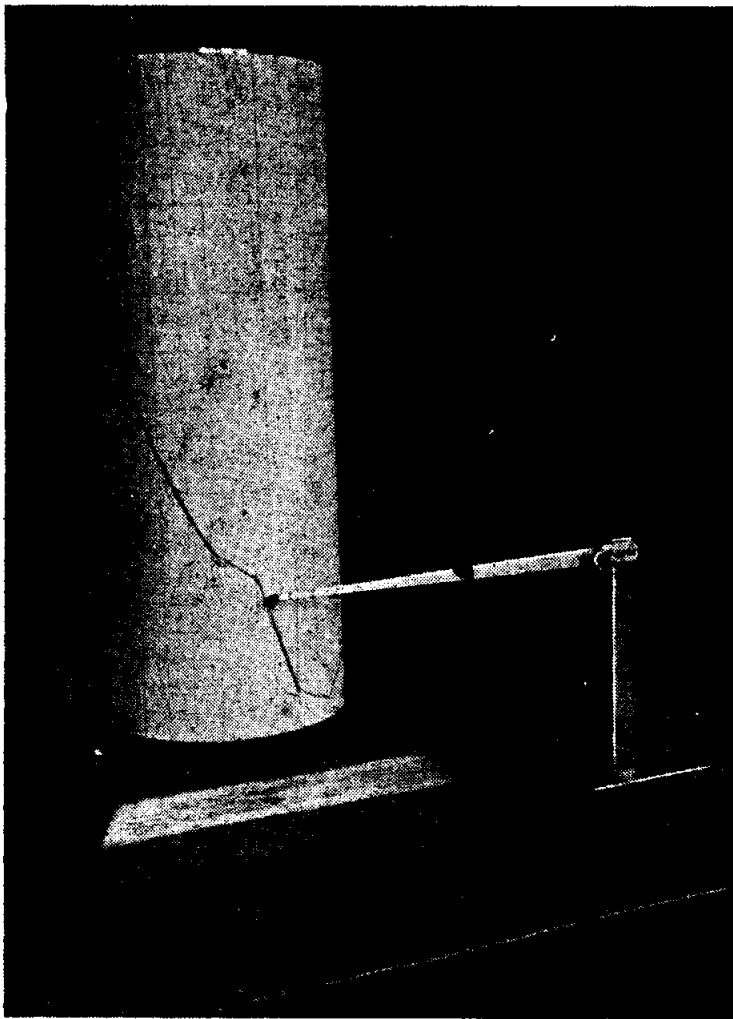


Fig. 16

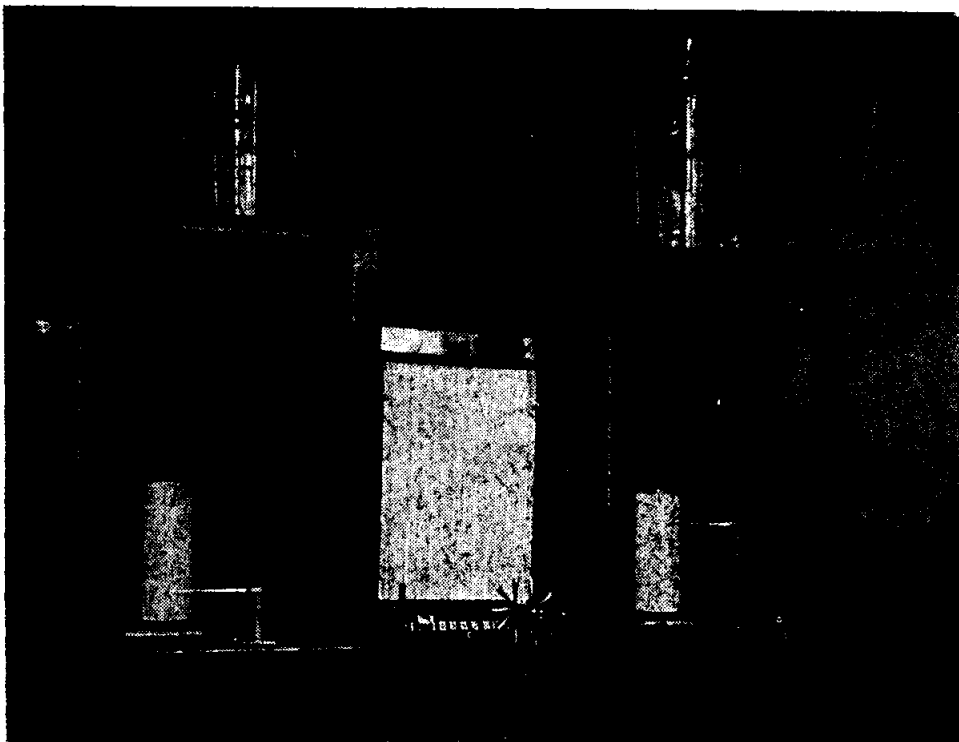


Fig. 17

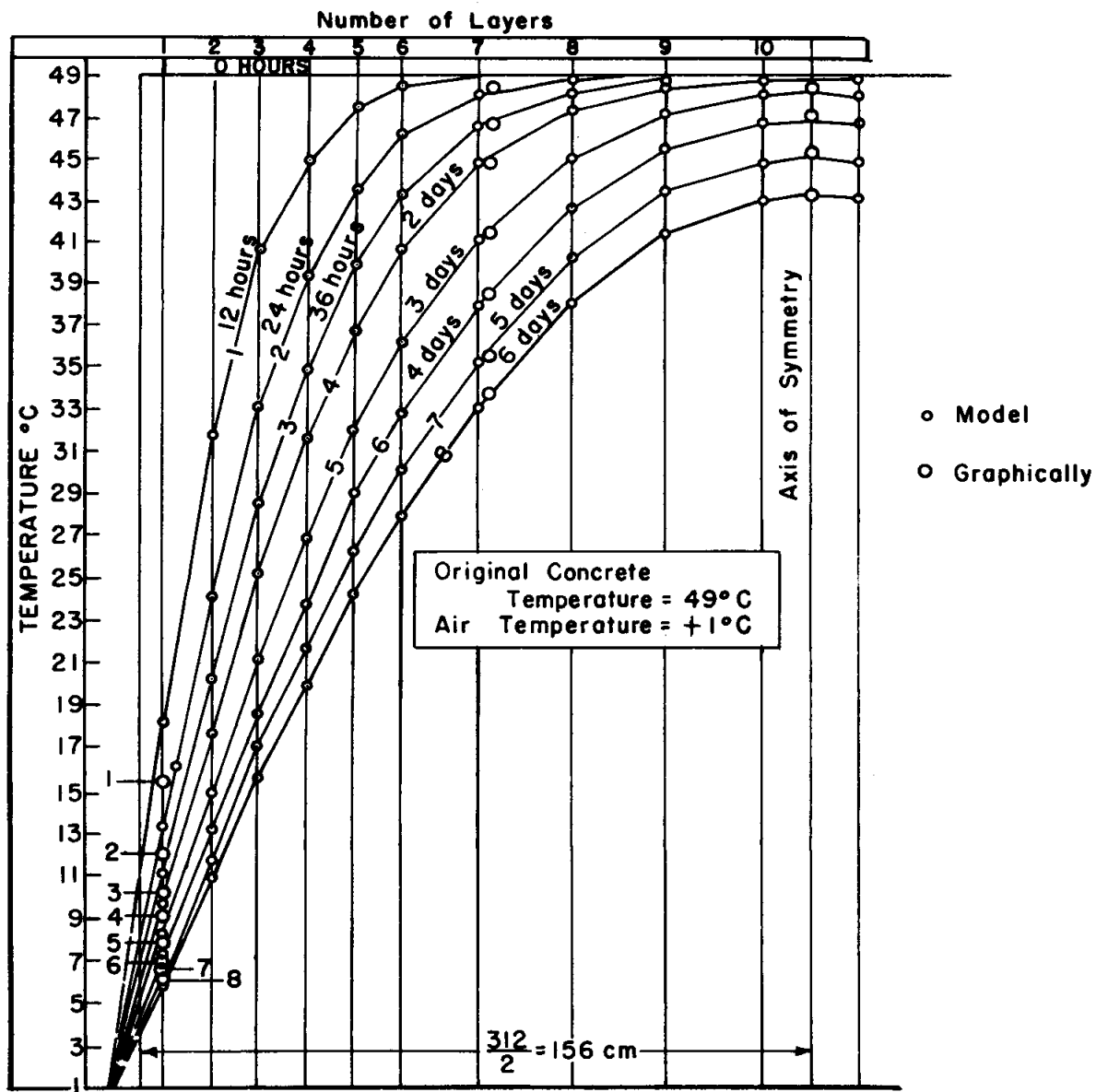


Fig. 18

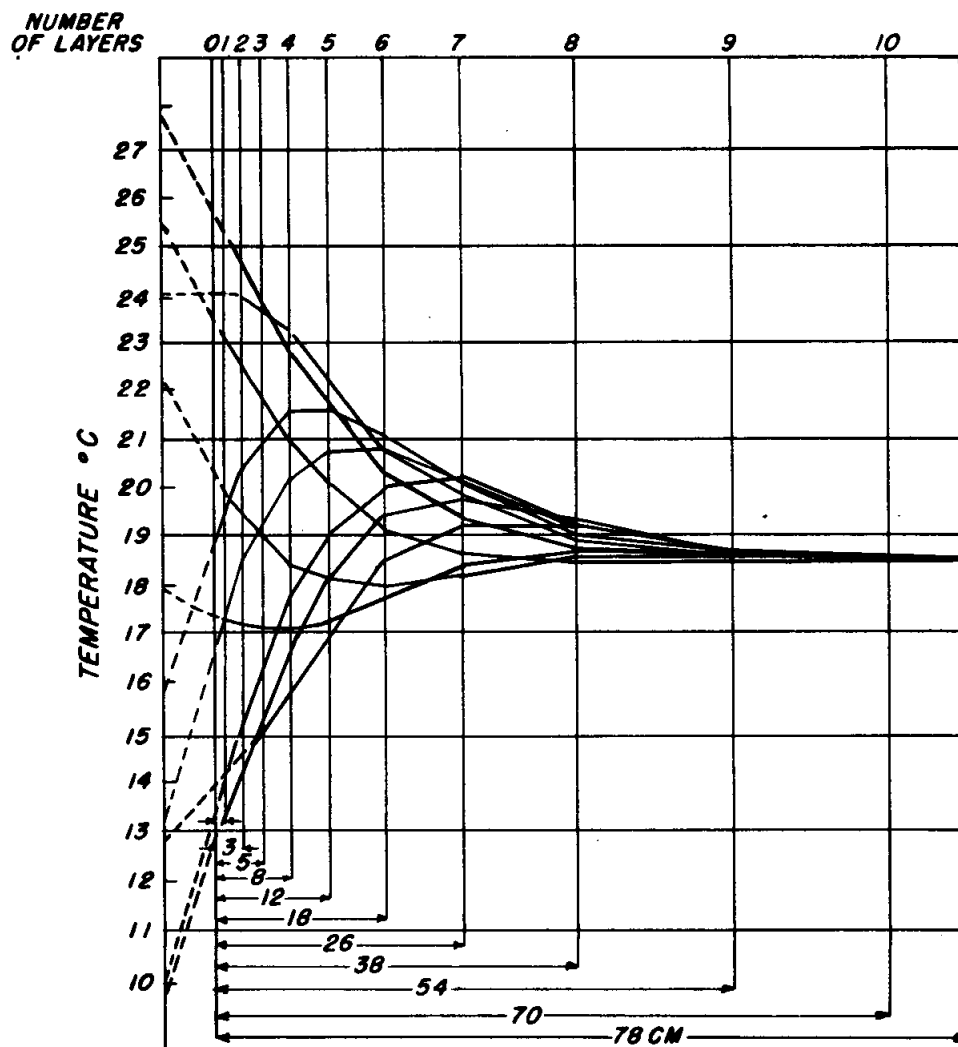


Fig. 19

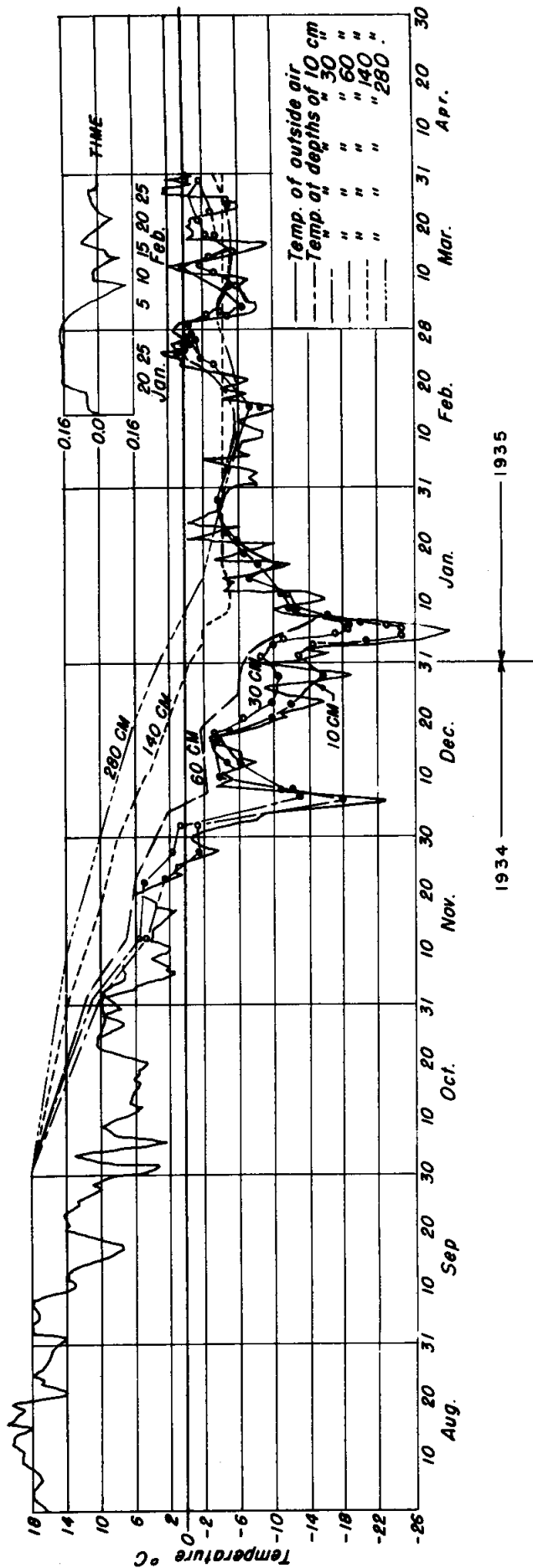


Fig. 20

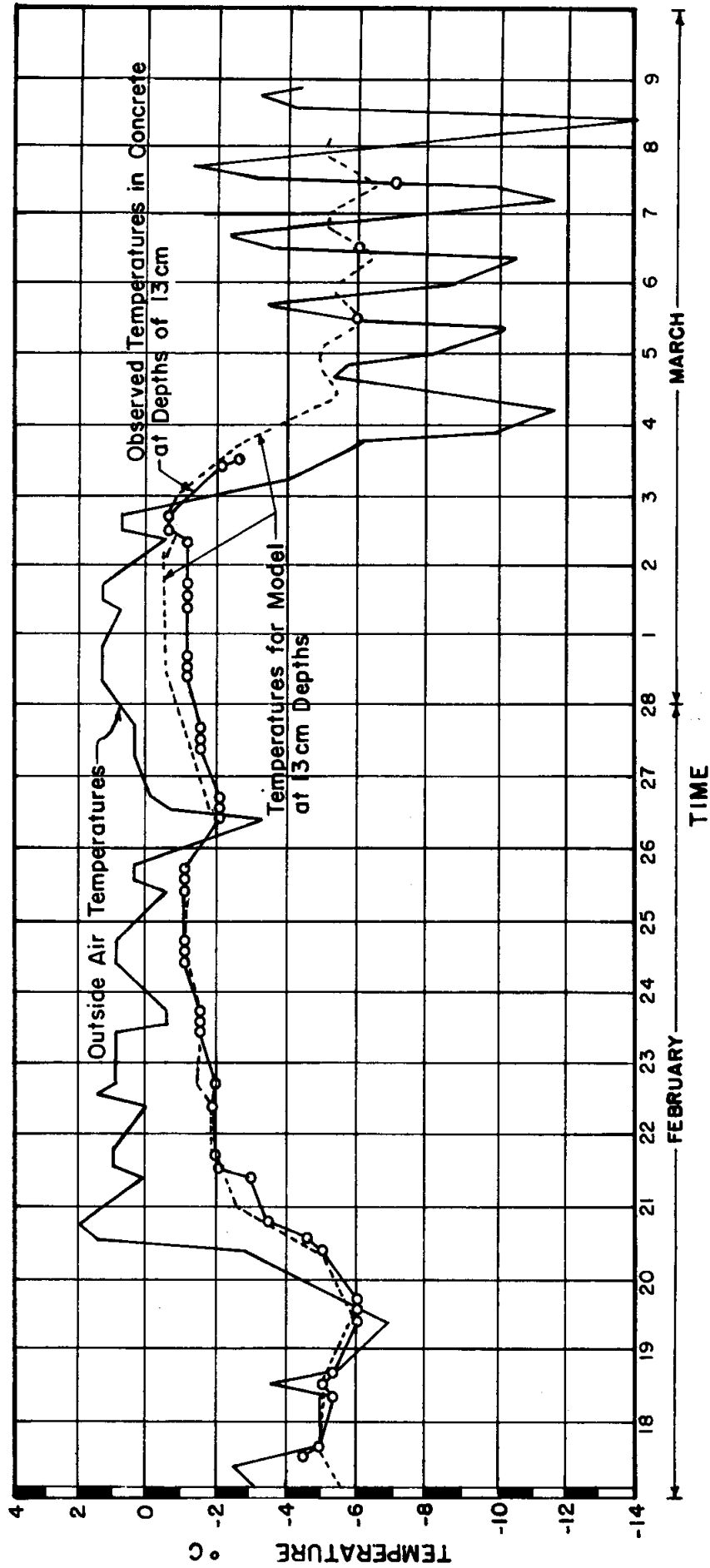
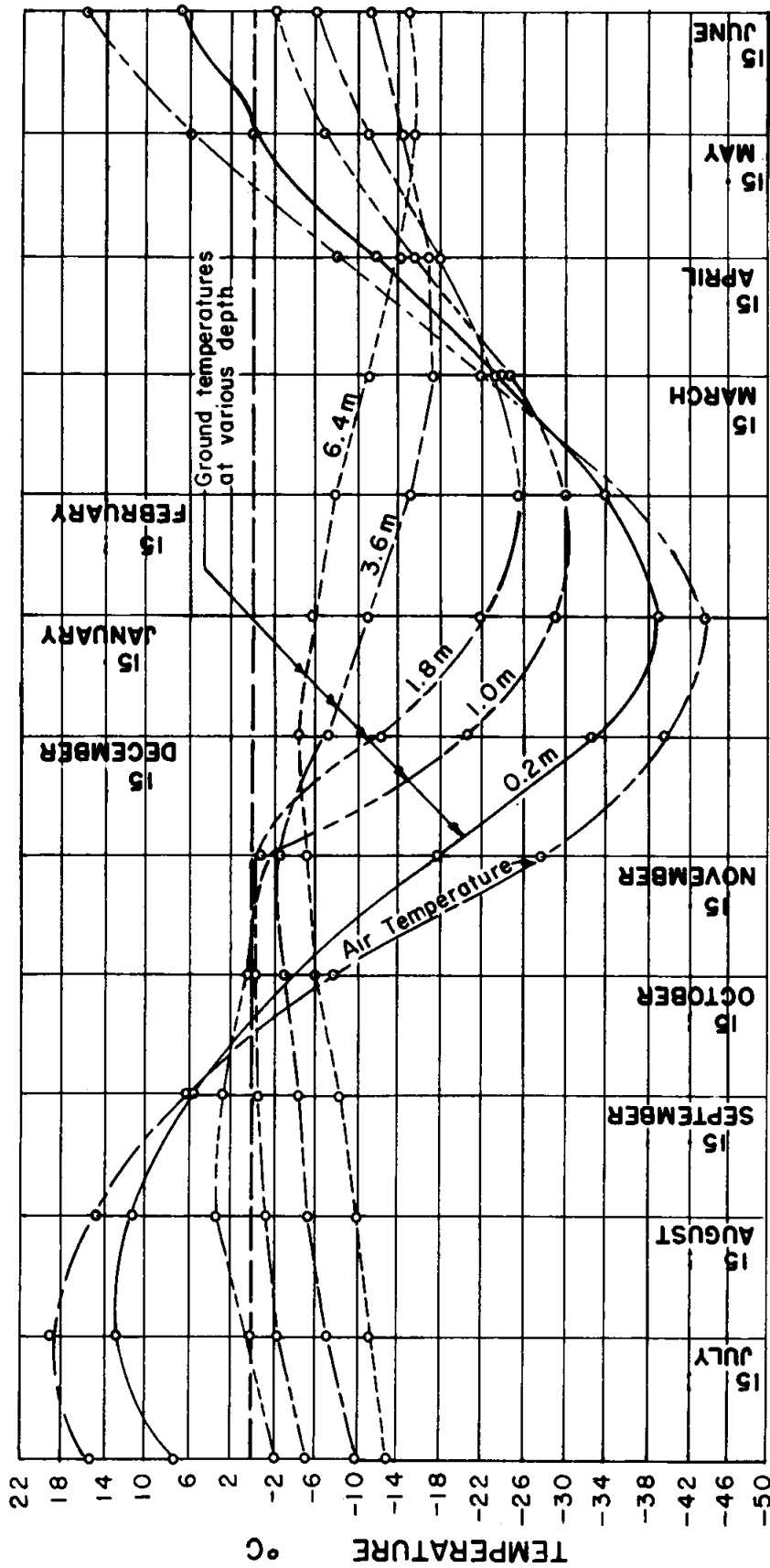


Fig. 21

in a concrete mass at a depth of 13 centimeters from the upper surface. The continuous line without the little circles shows the temperature of the outside air. The dotted line shows the temperatures in the concrete arrived at by using the machine, and the line with the circles alone shows the results obtained of actual temperatures in the concrete which were obtained by a mercury thermometer which was inserted into the concrete. The correlation of the results is very good. Figure 22 shows the results of machine calculations of the temperatures in moist soil (with a consideration of the latent heat of ice formation). These calculations are applicable to conditions of permafrost.

Figure 23 shows temperature curves calculated on the machine for a building with an attic. The curves show the temperature distribution at 2-hour intervals for given graphical values of outside and inside air temperatures.

Figure 24 shows the results obtained on the machine in determining the temperature of concrete under the action of the exothermic heat given off during hardening. Figure 25 shows the results obtained on the machine of the changing temperature of a layer of ice 50 centimeters thick under the action of the latent heat which is being given off by the freezing of water poured in a layer 1 centimeter thick on the surface of the ice. The temperature of the air is -20°C . Curves are shown corresponding to the calculated temperature after the elapsed time from 10 to 550 minutes, measured from the exact time that the water was poured onto the ice.



TIME

Fig. 22

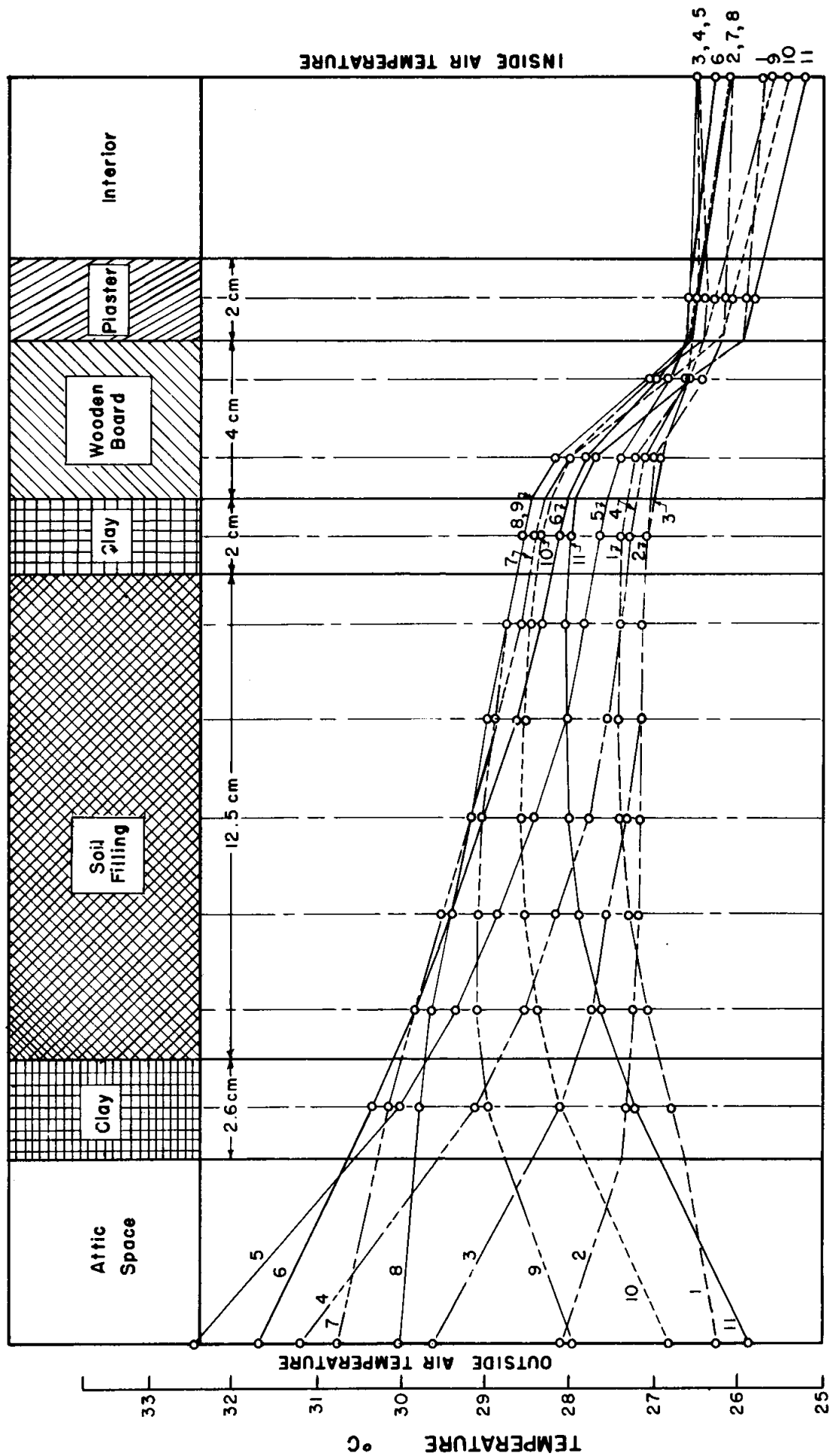


Fig. 23

NOTE:
Numbers show order in which readings
were taken every two hours.

NOTE:

The cooling of a concrete wall, thickness 1.5 m., taking into account the exothermic reaction of concrete. Insulator: wooden sheathing 5 cm. thick.

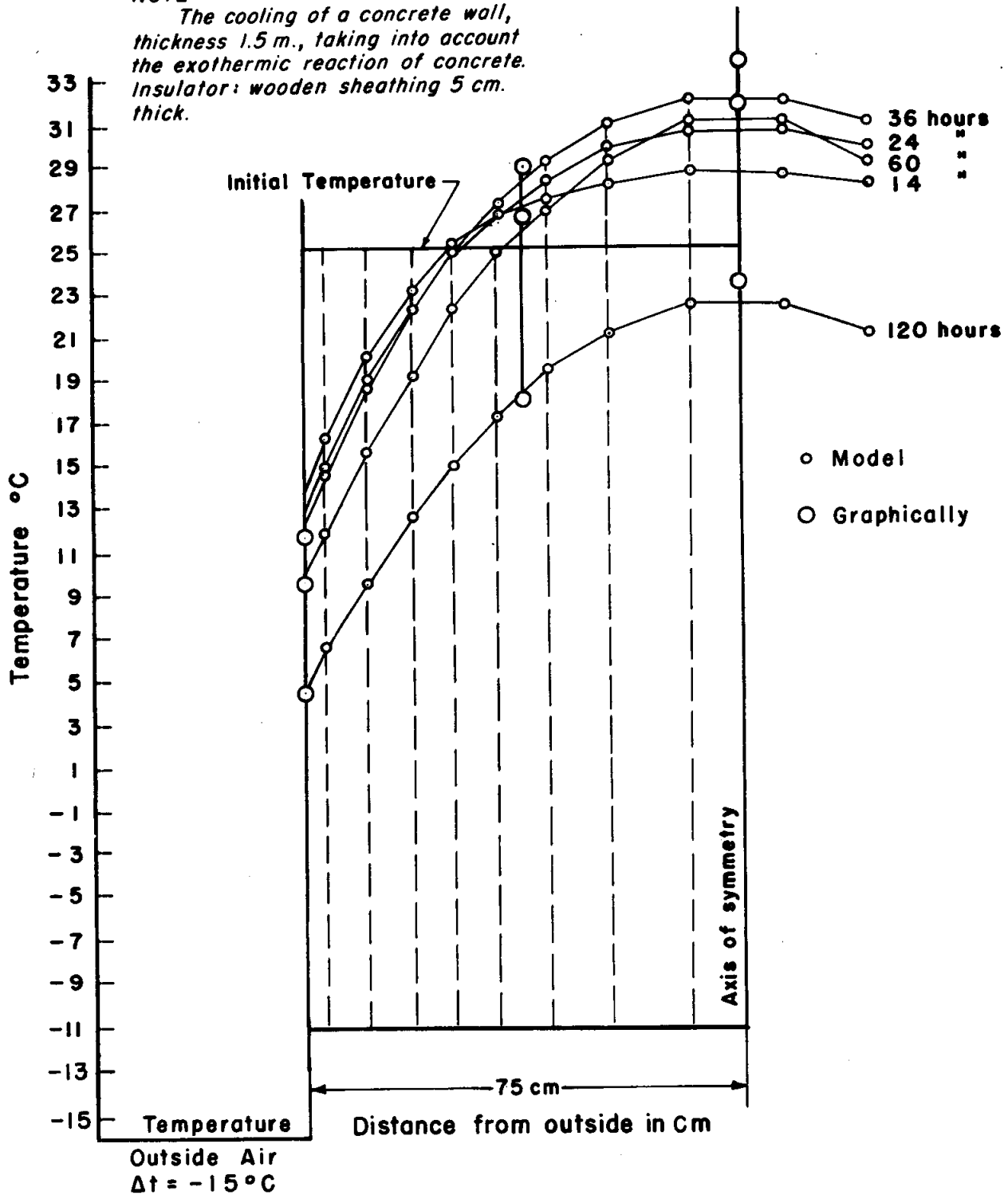


Fig. 24

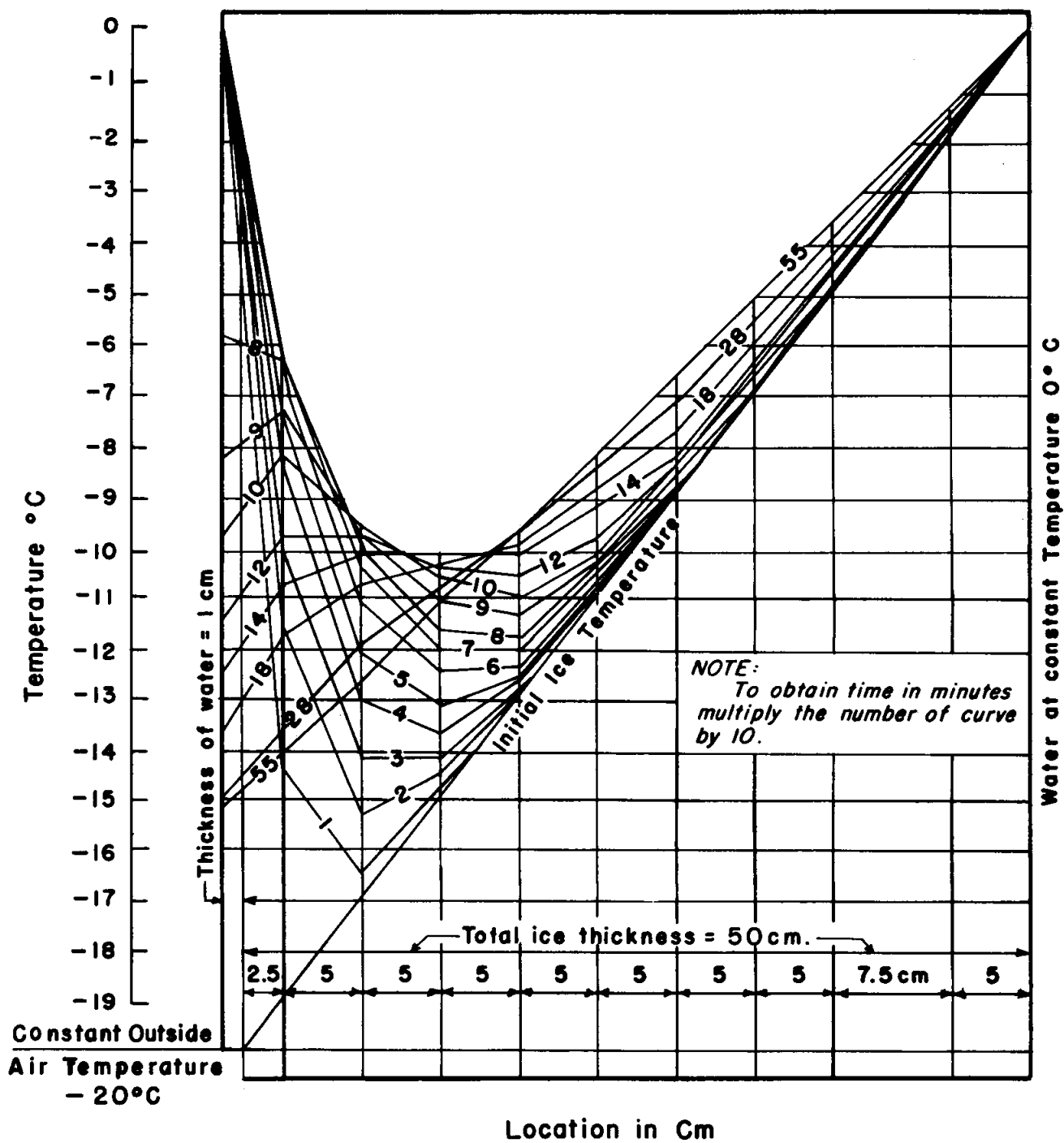


Fig. 25

The construction problem of a 2-dimensional machine (compare Fig. 5) is much more difficult than constructing a 1-dimensional model. The author, however, while working on the machine shown in Figures 8 through 17, had in mind the problems inherent in a 2-dimensional one. The desire to solve 2-dimensional problems explains the great effort of the author to expand the 1-dimensional model into a 2-dimensional one. It was supposed that a row of such 1-dimensional machines connected together would function as a 2-dimensional computer. However, such a model was made and it was evident that this construction was too unwieldy and inconvenient. The further building of a 2-dimensional model in the Institute of Roads and Construction, NKPS, was held up in 1937 because of certain conditions unfavorable for the Institute. In 1937 on the order of Glavstroiprom, the main construction center NKTP, preliminary work was done for the creation of a laboratory (which would contain a 2-dimensional model) at the Institute of Water and Hydraulics, NKTP. By dividing the functions of the various parts, the authors state that they simplified the construction of the machine. The size of one section is about 40 centimeters in width and with a depth of about 90 centimeters. A row of such sections makes up the machine. All the receptacles are connected into the machine separately and are situated at the back. In the common net, there are only the indicators of these receptacles, the collectors. The whole network and the whole control board are brought out forward. The change in cross-sectional areas of the receptacles is made by the summation of a proper number of receptacles with areas chosen on the principle of unequal sizes. The hydraulic resistances are regulated

by metallic elements. This sort of system makes it possible without many difficulties to obtain different schemes of connections between receptacles. At the present time a trial section has just been finished in the Institute of Vodgeo. ("Vodgeo" means hydraulic and geological sciences.) Since the authors do not have the opportunity to show in detail the construction details of their machine, that were worked out in Vodgeo, they will limit themselves to including two photographs of these trial machines.

Figure 26 shows the main section of the 2-dimensional machine. Figure 27 shows the general view of the trial installation in Vodgeo. The displacements of the control receptacles along the given curves in time are obtained automatically.

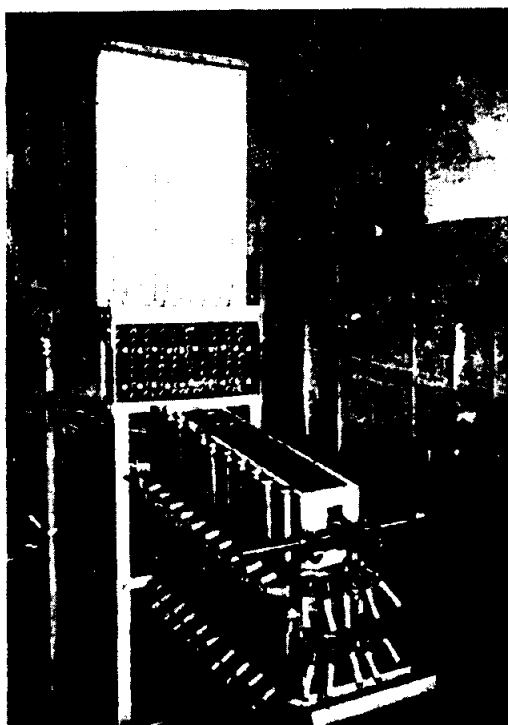


Fig. 26

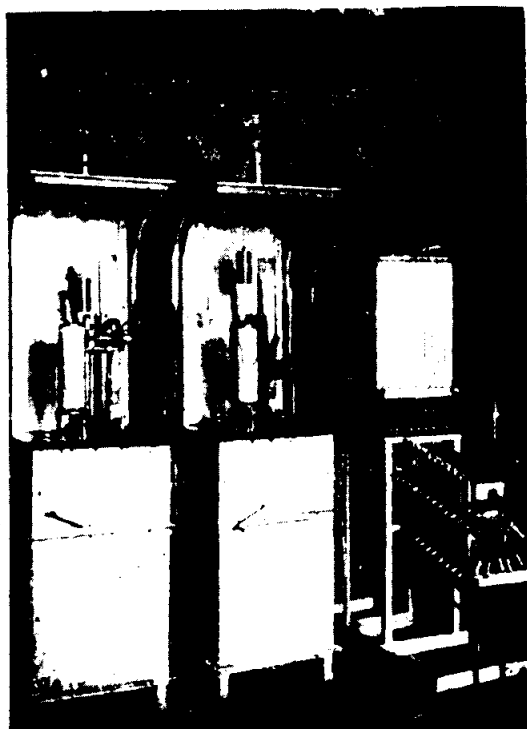


Fig. 27

